8 SCIENTIFIC HIGHLIGHT OF THE MONTH: "Ferromagnet/Superconductor Heterostructures" by Mariusz Krawiec

Ferromagnet/Superconductor Heterostructures

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Abstract

Some of the properties of the ferromagnet/superconductor proximity heterostructures are discussed. The particular emphasis is put on the physics of Andreev reflections featuring in unusual thermodynamic and transport properties of the system. These are: presence of the Andreev bound states, oscillatory behavior of the pairing amplitude, density of states and superconducting critical temperature, when the thickness of the ferromagnet is varied. They can produce spontaneous spin polarized currents flowing parallel to the interface, generating a magnetic field. They are responsible for a realization of a new state in such system, which is very similar to Fulde - Ferrell - Larkin - Ovchinnikov ($FFLO$) state predicted for an exchange split bulk superconductor. Some experiments, giving a rather surprising results may be explained in terms of $FFLO$ - Andreev bound states.

1 Introduction

Seventy years ago the proximity effect was observed experimentally [1] for the first time. Measuring the resistance of the normal metal (NM), placed between two superconductors (SC), one observed that superconductivity entered the normal metal, causing the resistance to vanish.

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Such a penetration of the superconducting properties into a normal state material is known now as a *proximity effect*.

The superconductivity is characterized by the order parameter $\Delta(\mathbf{r})$ which is related to the pairing amplitude $\chi(\mathbf{r})$ ($\Delta(\mathbf{r}) = g\chi(\mathbf{r})$, $g \neq 0$ only in SC). Physically the square of the pairing amplitude is the density of paired electrons. Unlike $\Delta(\mathbf{r})$, $\chi(\mathbf{r})$ can extend into a normal metal in contact with a superconductor, leading to the proximity effect. First theoretical studies, using these arguments, have been done in early sixties [2]. Roughly at the same time the Andreev reflection process was proposed [3]. According to it, an impinging electron (with energy less than SC gap) on the NM/SC interface is reflected as a hole, and the Cooper pair is created in superconductor. These processes allow for the transfer of $2e$ charge even though there are no quasiparticle states available. From this point of view the pairing amplitude can be regarded as a density of the correlated electron-hole pairs in the normal metal. So one can say that proximity effect and Andreev reflections are two sides of the same phenomenon. The effect has been extensively studied and is rather well understood by now [4]. The properties of such material under proximity are strongly affected. For example the system is able to carry the supercurrent, density of states posses a gap and the tunneling characteristics are modified.

When the normal metal is replaced by a ferromagnet (FM), another energy scale enters the problem, namely the exchange splitting which is related to the spin polarization of the electrons. It makes the the physics of the proximity systems much more rich [5, 6]. Such proximity effect between ferromagnet and superconductor is less well understood. Moreover, recently it has become possible to fabricate high quality FM/SC heterostructures [7]-[10] making these materials very attractive from point of view of scientific interest, as they allow for studying the interplay between magnetism and superconductivity [11] as well as of device applications in various areas of technology like magnetoelectronics [12, 13] for example.

It is widely accepted that ferromagnetism and superconductivity are two antagonistic phenomena, so one could expect that the proximity effect in FM/SC system should be suppressed. Indeed, the one can argue that in ferromagnet there are different numbers of spin-up (majority) $n_\uparrow$ and spin-down (minority) $n_\downarrow$ conduction channels, and due to the fact that incident and reflected particles occupy different spin bands, only a fraction $n_\downarrow/n_\uparrow$ of majority particles can be Andreev reflected [14]. Such suppression of the Andreev conductance has been observed experimentally in the structures consisting of metallic ferromagnets and classical (BCS) superconductors [15, 16] as well as in the colossal magnetoresistance materials in contact with high-T_c (d-wave) superconductors [17].

On the other hand if an exchange field acts on the Cooper pairs, one would expect that either it is too weak to break the pair, or it leads through the first order phase transition to the normal (ferromagnetic) state. However when a Cooper pair is subjected to the exchange field, it acquires a finite momentum and for certain values of the exchange splitting a new superconducting state is realized, known as *Fulde - Ferrell - Larkin - Ovchinnikov* ($FFLO$) state [18, 19]. Interestingly such state features a spatially dependent order parameter.

Similar oscillations of the pairing amplitude have been predicted [20]-[23] in ferromagnet/superconductor proximity systems. It turns out that these oscillations are responsible for the oscillatory behavior of the SC critical temperature T_c, first experimentally observed by Wong *et al.* [24], and the density of states [25] as the thickness of the FM slab is varied. In fact, the oscillations of the T_c in FM/SC multilayers can be also explained in terms of the effective π-junction behavior [21]. It was shown that at specific FM thickness the Josephson coupling between two SC layers can lead to a junction with an intrinsic phase (of the order parameter) difference $\delta\varphi = \pi$, which exhibits a higher T_c than the ordinary one ($\delta\varphi = 0$). The π-junction effect has been originally proposed by Bulaevskii *et al.* [26] to arise in the tunnel barriers containing magnetic impurities. Later on it has been shown that π-junction may exist in both ferro- and antiferromagnetic/superconductor multilayers [27]. It was also suggested that the π-junction can be realized in high-T_c superconducting weak links [28], where the SC order parameter changes its sign under $\pi/2$ rotation. This has tremendous consequences as it leads to many important effects [29, 30], like: the zero energy Andreev states, zero-bias conductance peaks, large Josephson current, time reversal symmetry breaking, paramagnetic Meissner effect and spontaneously generated currents.

From the point of view of the present paper the important issue is the formation of the Andreev bound states in FM/SC proximity system. The Andreev states arise due to the fact that the quasiparticles of the ferromagnet participating in the Andreev reflections move along closed orbits. Such states have been first studied by de Gennes and Saint-James [31] in the insulator/normal metal/superconductor ($I/NM/SC$) trilayer. The energies of these states are always smaller that SC gap Δ and symmetrically positioned around the Fermi level. They strongly depend on the geometry of the system as well as on the properties of the interfaces. In high-T_c (d-wave) superconductors, these states can be shifted to zero energy, due to the specific form of the symmetry of the order parameter [32], thus indicating π-junction behavior in the system. Naturally, such Andreev states can also arise in the $I/FM/SC$ heterostructures. Moreover, it is possible to shift the energies of these states changing the exchange splitting, as was first demonstrated by Kuplevakhskii & Fal'ko [33]. In turn, by properly adjusting the exchange splitting the position of the Andreev bound states can be moved to the Fermi energy. The system under such circumstances behaves like that being in the π-junction phase as the spontaneous current is generated [34]. However the physical origin of these states is quite different.

In the present paper the properties of the FM/SC heterostructures are discussed in terms of the $FFLO$ state and Andreev bound states. In some situations the state, which has properties of both the $FFLO$ and the π-junction, is realized, leading to various interesting and unexpected phenomena. The present paper is not intending to be a review article, as many issues have been omitted, but a brief look at the results of the recent experiments on FM/SC heterostructures from point of view of the $FFLO$ - Andreev bound states physics.

The paper is organized as follows: In the Section 2 some notable experimental results are presented. Sections 3 and 4 refer to the Fulde - Ferrell - Larkin - Ovchinnikov state in the exchange split bulk superconductor and FM/SC heterostructure respectively. The origin and the nature of the Andreev bound states is explained in Sec. 5, while their realization in FM/SC is studied in Sec. 6. Some recent results regarding the generation of the spontaneous currents in those systems are presented in Sec. 7 and finally, Sec. 8 contains summary and some concluding remarks.

2 Surprising experimental results

Since pioneering experiments on spin polarized tunneling in superconductors [35] a large experimental and theoretical effort has been done to understand the interplay between ferromagnetism and superconductivity in the FM/SC hybrid structures. According to the conventional point of view the proximity effect in FM/SC system should be very short ranged due to the destructive nature of the ferromagnetism on superconductivity. However some of the experiments seem to be in contradiction to this conventional wisdom.

2.1 Conductance and magnetoconductance

Lawrence and Giordano [36] measured the magnetoresistance of $Pb/Ni/Pb$ and $In/Ni/In$ structures and found it to be two orders of magnitude larger than it was predicted by theory. The effect has been attributed to an 'unusual' proximity behavior. Petrashov *et al.* [37] studied the proximity-induced conductance on the FM side of the hybrid FM/SC (Ni/Al) structures and again difference between experiment and theory was two orders of magnitude. Moreover they have observed new peaks in the differential conductance on the superconducting side, giving a clear evidence of a strong mutual proximity effect. Very long range proximity effect with new peaks in magnetoresistance of the ferromagnetic (Ni) wires deposited on SC (Al) have also been observed and explained analyzing the topologies of actual Fermi surfaces in ferromagnet [38]. Another experiment on Co/Al nanostructure [39] showed a drop in resistance of the Co wire, while differential conductance data suggested decay length for proximity effect to be about 180 nm, which is an order of the magnitude larger than that expected from the exchange field of the ferromagnet. Similar conclusions have been reached by Petrashov *et al.* [8] in the case of SC (Al) islands deposited on FM (Ni) structure. All above transport experiments support the very long range proximity effect scenario in such FM/SC heterostructures. However, there are also alternative explanations due to the spin accumulation effect at the FM/SC interface [40] or anisotropic magnetoresistance [41].

2.2 Interface properties

The important problem in the transport experiments is the quality of the interface. The issue has been raised by Aarts *et al.* [42] suggesting that interface transparency for the Cooper pairs strongly depends on the pair breaking effect in FM layer and hence on the exchange field. Indeed it has been shown theoretically [43] that interface resistance can, not only quantitatively but also qualitatively, modified transport properties of the FM/SC heterostructures consisted of either BCS or high-T_c superconductors. For the $Nb/Al/Gd/Al/Nb$ junction it has been confirmed experimentally that main contribution to the resistance comes from the interface scattering (unlike the NM/SC case, where the bulk scattering is very important) by Bourgeois *et al.* [44]. Moreover, the explanation of the measured conductance of the Co/Pb nanocontacts by Soulen *et al.* [15], needs some modifications of properties of the interface even though a band structure of the materials is taken into account [45].

2.3 Transition temperature - FM/SC multilayers

Since it is easy to measure, most of the experimental efforts has focused on the SC transition temperature of the FM/SC multilayers, sometimes showing surprising and intriguing results. Wong *et al.* [24] for the first time has observed oscillations of the SC transition temperature T_c as a function of the FM slab thickness in Fe/V multilayers. The result remained in a contradiction with the conventional point of view of the destructive nature of the ferromagnetism on superconductivity. This curious behavior has been attributed to the formation of an effective π-junction in such structure [21]. At certain thicknesses of the FM layer the state with the phase of the order parameter across the FM layer equal to π is realized rather than usual 0 state. So when the thickness of the FM is varied, the system chooses the state with the higher T_c, thus switching between 0 and π-phase.

The oscillations of T_c have also been seen in structures consisted of other materials, usually metallic ferromagnets and superconductors like Gd/Nb [46], Co/Nb and Co/V [47]. The oscillating behavior also have been found in $CuMn/Nb$ (spin-glass/superconductor) multilayers [48] and even in an insulating ferromagnet/superconductor (GdN/NbN) system [49]. The example of such behavior of the T_c as a function of the FM thickness d_{Gd} is shown in the Fig. 1, taken from Ref. [46]. The system consisted of Gd/Nb multilayers sputtered on Si substrate. The

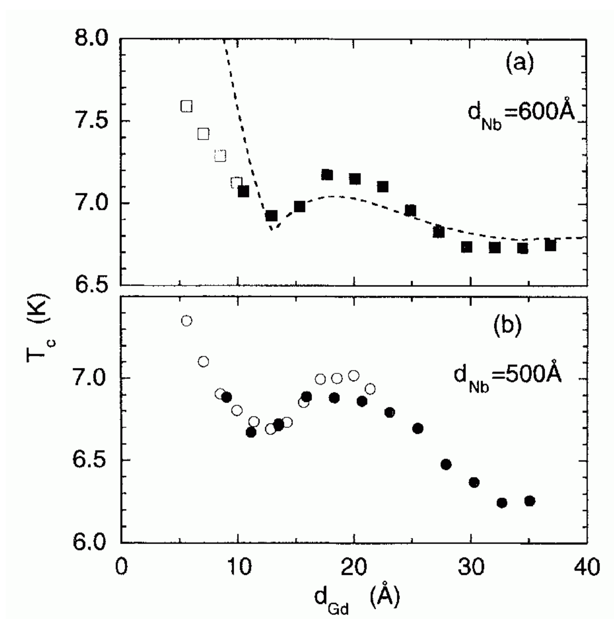

Figure 1: Superconducting transition temperature T_c vs FM thickness d_{Gd} in Gd/Nb multilayers with (a) $d_{Nb} = 600$ $\AA$ and (b) 500 $\AA$. Different symbols correspond to different sample series. Data reproduced from Ref. [46].

critical temperature has been determined from resistivity measurements by standard four-probe technique as well as from ac susceptibility.

Subsequent experiments [50]-[52] have shown no oscillations of T_c. In particular, Koorewaar *et al.* [50] studied T_c in Fe/V multilayers and concluded that superconductivity is decoupled even by ultrathin Fe layers. Strunk *et al.* explained monotonic behavior of T_c by short penetration depth of the Cooper pairs in Gd/Nb system. No oscillations have been observed in VFe/V multilayers [42] due to the interface scattering. Verbanck *et al.* [52] observed also monotonic behavior of T_c, however at certain value of the Fe thickness of the Fe/Nb system T_c vs d_{Fe} curve showed sudden drop. This effect is due to the non-magnetic behavior of thin Fe slab, while for larger Fe thicknesses the Nb layers are decoupled by Fe which became ferromagnetic.

In fact one can explain both behaviors: the oscillations of T_c and their lack by including spin-orbit coupling into theory [53]. While the exchange filed in FM is responsible for oscillations, the spin-orbit coupling tends to suppress them. So in general the π-junction scenario [20, 21, 54] has been wide accepted as an explanation of the oscillatory behavior of the superconducting transition temperature in FM/SC multilayers.

2.4 Transition temperature - FM/SC/FM trilayers

Some of the experiments performed on $FM/SC/FM$ trilayers [55]-[57] and even FM/SC bilayers [57] showed also oscillating nature of T_c. Such curious behavior cannot be explained with help of the π-junction, because simply there is only one SC layer. So other explanations have been proposed. Mühge *et al.* [55] studied $Fe/Nb/Fe$ trilayer and the observed oscillations of T_c explained in terms of a rather complex behavior of the magnetically "dead" Fe layer near the interface. They argued that at low d_{Fe} the layer is a strong pair breaker due to induction of the repulsive electron-electron interaction by Fe d-levels. Further, at certain d_{Fe} T_c start to raise because the FM order in Fe is nucleated leading to a Zeeman splitting of the d-states and reducing the repulsive interaction. And finally, the maximum and further decrease of T_c occur due to the domination of the direct exchange splitting as the thickness increases. Tagirov *et al.* [56] observed not only oscillations of T_c but also re-entrant effect in $Fe/V/Fe$ trilayers. At certain d_{Fe} the superconductivity is destroyed, and as the FM thickness is increased further, the SC state is recovered. The other experiments showed non-monotonic behavior of T_c in $Fe/Pb/Fe$ [57] trilayers and even bilayers: Fe/Pb [57] and $CuNi/Nb$ [58]. Certainly such behavior is a clear indication of an unconventional, propagating state in the ferromagnet in proximity to the superconductor. Finally, there were other works on similar structures where the oscillations have not been found [59]. Also result was negative for $Fe/Pt/Nb$ ($FM/NM/SC$) trilayers [60] due to the strong influence of the non-magnetic (Pt) spacer.

Subsequent theoretical works [22, 23, 61]-[64] showed that oscillations of the superconducting transition temperature could be also explained in terms of the Fulde - Ferrell - Larkin - Ovchinnikov ($FFLO$) state [18, 19]. Moreover, the $FFLO$ scenario allows for a non-monotonic behavior of T_c even for bilayers [22, 23, 56]-[63]. So this makes the $FFLO$ scenario a natural explanation of the oscillatory behavior of T_c in the FM/SC heterostructures. The fact that in some experiments oscillations of T_c have been seen [24, 46]-[49, 55]-[57] and in other have not [50]-[52, 59, 60] can be due to the properties of the FM/SC interfaces (transparency) [23, 63] as well as due to the disorder [22, 23, 62]. Such scenario is consistent with the recent experiment [65], where T_c in $Nb/PdFe/Nb$ has been studied, and the T_c showed its oscillatory behavior or did not, depending on the iron concentration. However the results cannot rule out the π-junction behavior, as there are two SC layers in this system.

There is another aspect supporting the realization of the $FFLO$ state in FM/SC structures. Despite the oscillations of the T_c, sometimes the sudden kink at certain value of the FM thickness was observed [66, 56]. This effect cannot be explained neither in terms of the π-junction behavior nor by usual $FFLO$ state, in which the pairing amplitude varies only in the direction perpendicular to the interface. However it turns out that at certain conditions $3D$-$FFLO$ state, featuring in spatial dependence of the pairing amplitude also along the interface, can be realized [67, 6]. In this case the system chooses the lower energy ground state and is switched between usual $1D$ and $3D$ $FFLO$ state producing sudden jump of the critical temperature. Of course, the oscillations of T_c, their lack and reentrance of the SC can be also realized within this scenario.

Recently it has been predicted theoretically [13, 20, 62, 63] that the $FM/SC/FM$ systems should exhibit different SC transition temperatures depending on the direction of the magnetization in the FM layers. In particular it can be switched between superconducting and normal states in antiparallel and parallel magnetization configurations respectively. This effect has been observed experimentally [69].

2.5 Critical Josephson current

The question of the π-junction behavior has been also addressed theoretically, studying the Josephson critical current through ferromagnetic spacer [20, 70]-[76]. It has been predicted [20], that Josephson critical current I_c should exhibit an anomalous FM thickness dependence while switching between 0 and π-phase. In particular the amplitude of I_c should go to zero at the transition point ($d_{FM} = d_{FM}^{crit}$). Similar behavior is expected for the system with properly adjusted FM thickness ($d_{FM} = d_{FM}^{crit}$) when the temperature is changed (see however [76]). First experimental evidence of the π-junction has been given by Ryazanov *et al.* [77], where the critical Josephson current in the $Nb/CuNi/Nb$ trilayer has been measured as a function of the temperature (see Fig. 2). They found that I_c vanishes at the transition point. In fact I_c does not have to be zero, at this point due to the higher order Josephson coupling [76]. Another experiment [78], measuring I_c in $Nb/Al/Al0/PdNi/Nb$, but as a function of the FM layer thickness, also confirmed realization of the π-junction state in this system.

2.6 Tunneling and the density of states

One of the features of the π-junction state is the zero-bias conductance peak ($ZBCP$) [29]. In fact such $ZBCP$ has been observed very recently in $Nb/FeSi/Nb$ tunnel junction [79]. Interestingly, it turns out that, due to the polarization of FM and the Fermi wave vector mismatch between FM and SC regions, the $ZBCP$ can also emerge when the conductance of the FM/SC bilayer is calculated [43]. However, in Ref. [43] a step like function for the pairing potential $\Delta(\mathbf{r})$ has been assumed, thus neglecting the proximity effect. If $\Delta(\mathbf{r})$ is calculated self-consistently, we believe that $ZBCP$ can also emerge without the Fermi wave vector mismatch due to the $FFLO$ Andreev bound states in the system [80, 81].

The density of states (DOS) at the Fermi energy shows the oscillatory behavior as a function of the thickness of the FM layer [82]-[85, 34, 80, 81]. The oscillations of DOS have been

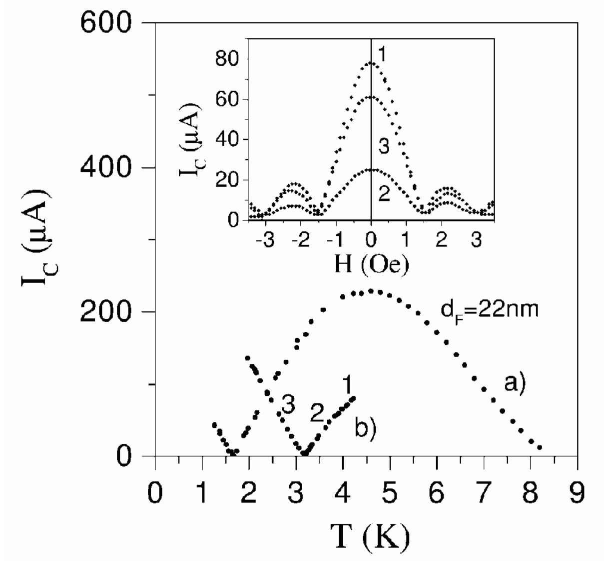

Figure 2: Critical current I_c in the $Nb/CuNi/Nb$ heterostructure as a function of temperature T for two junctions with $Cu_{0.48}Ni_{0.52}$ and $d_{FM} = 22\ nm$. Inset: I_c versus magnetic field H for the temperatures around the crossover to the π state as indicated on curve b): 1 - $T = 4.19\ K$, 2 - 3.45 K and 3 - 2.61 K. Data reproduced from Ref. [77].

observed experimentally [25] in the planar tunneling spectroscopy of the $Al/AlO/PdNi/Nb$ heterostructure. The results are depicted in the Fig. 3. The experimental results have been quantitatively explained in terms of the $FFLO$ scenario and Andreev bound states [83, 82, 86]. However, additionally the effect of the finite interface resistance had to be taken into account [25, 83, 82, 86] in order to get the quantitative agreement with experiment [25]. On the other hand, the disorder due to the impurity scattering is of no importance in this system as the good fit has been achieved in the clean limit.

The other experiments regarding magnetic proximity effect [87], magnetic coupling through the SC spacer [88, 89] and experiments on colossal magnetoresistance materials in contact with high-T_c superconductors [10] have been omitted. The only those important from a point of view of the present paper, $FFLO$ Andreev bound states physics, have been discussed.

3 Superconducting electrons in an exchange field - FFLO state

It is well known that the exchange field tends to polarize the conduction electrons in a metal. Now the question raises what will happen if these electrons are Cooper paired. In other words what will be effect of the exchange field on superconductor. At zero temperature, naively one would expect that this field is either too weak to break the Cooper pairs, thus it leaves the SC state unchanged, or it produces a first order phase transition to the normal state. However, it turns out that for certain values of the exchange splitting a new supercondcuting depairing ground state, with both the Cooper pairs and unpaired electrons present, can be realized [18, 19]. This state is known as *Fulde - Ferrell - Larkin - Ovchinnikov* ($FFLO$) state and it evolves from

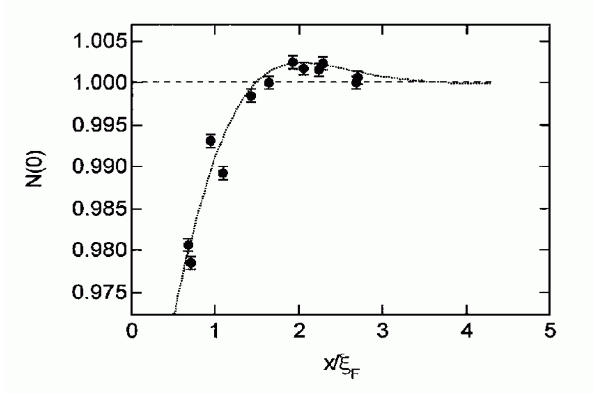

Figure 3: Tunneling conductance of the $Al/AlO/PdNi/Nb$ junction at zero energy vs the $PdNi$ thickness normalized by the coherence length ξ_{FM}. The data taken at $T = 300\ mK$ and $H = 100\ G$ are shown as solid symbols. The dotted line represents theoretical fit, while the dashed one denotes the transition from the 0- to the π-state. From [25].

the BCS, by the first order, to the normal state, by the second order phase transition, as the exchange energy is increased. Actually this is true only for the $3D$ superconductor with the spherical Fermi surface, as it was shown later [90] the phase diagram of such system strongly depends on dimensionality and thus properties of the Fermi surface. The phase diagram of the $3D$ BCS superconductor in an exchange field is shown in the Fig. 4. One can note that $FFLO$

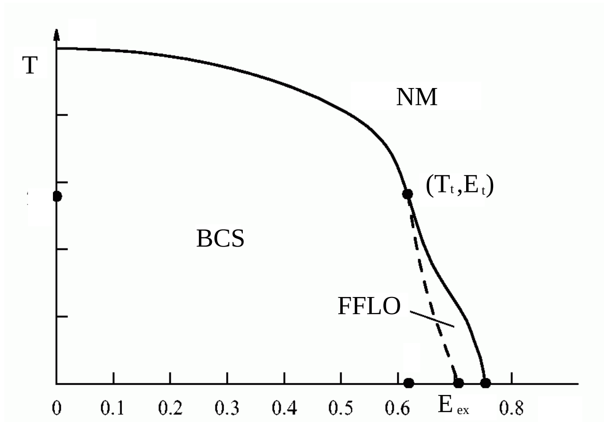

Figure 4: (T, E_{ex}) phase diagram of the BCS superconductor in the exchange field. E_{ex} is in units of Δ_0. (T_t, E_t) denotes tricritical point with $T_t = 0.56T_c$ and $E_t \approx 0.62\Delta_0$. Solid line denotes second order phase transition and the dashed one - first order. Reproduced form [6].

state is realized only in very narrow range of parameters.

The $FFLO$ state is characterized by spatially dependent order parameter corresponding to the non-zero center of mass motion of the Cooper pairs, i. e. $\Delta(\mathbf{r}) = \Delta_0 e^{i\mathbf{Qr}}$ [18] or $\Delta(\mathbf{r}) = \Delta_0 cos(\mathbf{Qr})$ [19] with $\mathbf{Q}$ depending on the exchange splitting E_{ex} and the Fermi velocity v_F ($|\mathbf{Q}| = 2E_{ex}/v_F$). At this point it well to raise an issue regarding the physical origin of such oscillations. Imagine a Cooper pair subjected to the exchange field E_{ex}. Upon acting of E_{ex} the pair is not an eigenstate any more. Moreover, due to the exchange field, the spin up electron in the Cooper pair lowers its potential energy, while the spin down electron raises it. On the other hand, the total energy has to be conserved for each electron, so the spin up (down) electron must increase (decrease) its kinetic energy (see upper part of Fig. 5). And thus the Cooper pair

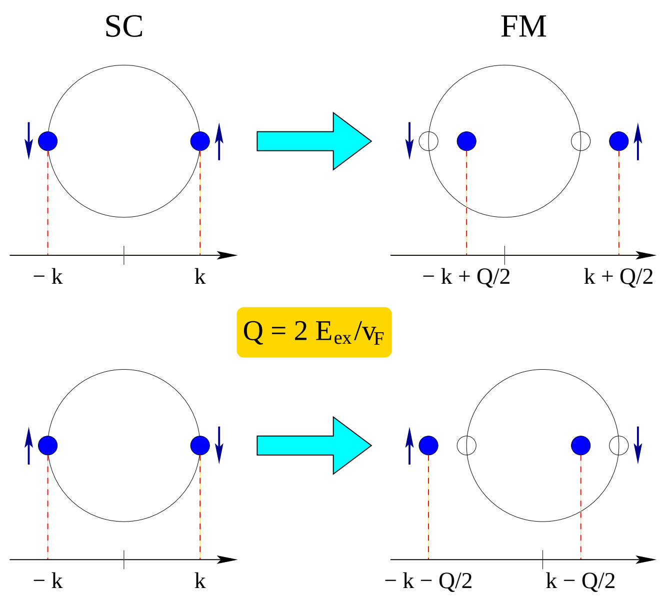

Figure 5: Cooper pairs in superconductor (SC) and in superconductor in an exchange field (FM). Adapted from Ref. [22].

acquires a center of mass momentum $\mathbf{Q}$. Similar is true for the pair with its spins interchanged, shown on the bottom of Fig. 5.

The $FFLO$ state has some polarization due to the unpaired electrons and hence it displays almost normal Sommerfeld specific heat and single electron tunneling characteristics [18, 91]. The non-zero value of pairing $\mathbf{Q}$ also gives rise to the unusual anisotropic electrodynamic properties [18, 91]. Another feature of this state is a current flow in the ground state. It consists of two parts. One is due to the unpaired electrons, which tend to congregate at one portion of the Fermi surface, and the other one is a supercurrent generated by the non-zero value of the pairing momentum. Both currents flow in opposite directions, thus cancel each out, so the Bloch theorem (no current in the ground state) is satisfied. Finally, it turns out that this state is very sensitive to both potential scattering τ_f and spin-orbit scattering τ_{so} and is destroyed when the product of the mean free time and BSC energy gap Δ_0 is equal to unity [91]. This may be the reason why there is no unambiguous experimental evidence of the $FFLO$ state in bulk materials. However this state might already be seen in some quasi-two-dimensional organic conductors (see e.g. [92]).

4 FFLO state in FM/SC heterostructures

It is widely accepted that $FFLO$ state in natural way can emerge in the ferromagnet/superconductor heterostructures. One can imagine that the Cooper pair from SC enters into FM and experiences the exchange field [22]. In this case, the pair is not a eigenstate of FM, so it becomes an evanescent state, exponentially decaying over the distance of the normal metal coherence length, as in the case of usual proximity effect [4]. Moreover, due to the exchange splitting, it acquires a center of mass momentum $\mathbf{Q}$. So the wave function of the pair (or pairing amplitude $\chi(\mathbf{r})$) receives a spatial modulation, similarly as in the bulk SC pairing potential $\Delta(\mathbf{r})$ does [93]. In general, one has to take into account both cases, shown in the upper and lower parts of the Fig. 5. According to this picture, the 'upper' Cooper pair wave function acquires the momentum equal to $\mathbf{Q}$, while the 'lower' one $-\mathbf{Q}$, so the modulation factor is $cos(\mathbf{Qr})$, like for the scenario proposed in Ref. [19]. This is true for the Cooper pair moving perpendicular to the FM/SC interface. Now if one assumes all possible angles of incidence for this pair, the spatial modulation of the pairing amplitude in FM is given by $sin(x/\xi_{FM})/(x/\xi_{FM})$ [22], where x is the distance from the interface, while $\xi_{FM} = \hbar v_{FM}/E_{ex}$ is the FM coherence length. Typical example of such behavior, obtained numerically [80], is shown in the Fig. 6. It turns out, that, unlike in the

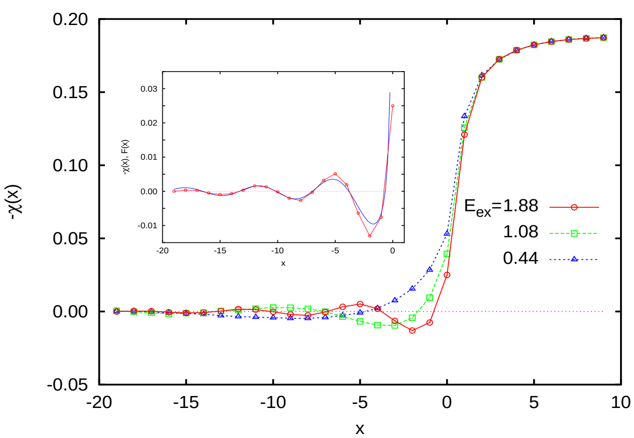

Figure 6: The pairing amplitude vs distance from the interface x for values of the exchange field indicated in the figure, in units of $t = W/8$, where W is a bandwidth. Inset: comparison of the numerical results with the analytical formula $\chi(x) \propto sin(x/\xi_{FM})/(x/\xi_{FM})$ for the exchange field $E_{ex} = 1.88$. From Ref. [80].

bulk SC, these oscillations are obtained for a wide range of the parameters [22, 94, 84, 34, 80], so it means that $FFLO$ state can be more easily realized in FM/SC heterostructures. Indeed, such oscillations have been indirectly seen experimentally by Kontos *et al.* [25]. Similar oscillations have also been predicted [95, 96] for a superconductor with d-wave symmetry of the order parameter. Additionally it has been observed to be generating a p-wave component of the SC order parameter near the interface [95].

The important issue is the question of disorder, as it is known to be very destructive for the bulk $FFLO$ state [91]. Namely, the $FFLO$ phase disappears when $1/\tau_f$ or $1/\tau_{so} > \Delta_0$. The situation is quite different in FM/SC proximity system, where superconductivity and magnetism are spatially separated. Of course, the effect of both the elastic potential and the spin-orbit scattering is to lower the effective period of the oscillations of the pairing amplitude as well as to introduce additional its decay [22]. In particular, in the dirty limit (strong disorder), the oscillations are damped on the same length scale on which they oscillate, so the $FFLO$ state is suppressed. However, the mean free time (between successive scattering events) is proportional to $1/E_{ex}$ rather than $1/\Delta_0$, as the energy scale in ferromagnet is set by the exchange splitting only (the SC gap parameter Δ_0 vanishes in FM). So one can say that the disorder is less destructive for the $FFLO$ state in this case. Moreover, the stronger ferromagnet is, the larger disorder is required to destroy the $FFLO$ state. This is the reason why the $FFLO$ state has been observed experimentally in FM/SC heterostructures even the ferromagnet was an alloy, like in [58, 65, 25].

By taking into account the effect of the elastic scattering (non-magnetic disorder) in the $FFLO$ state, one can theoretically describe all behaviors of SC transition temperature T_c observed experimentally: oscillations of T_c or their lack as well as the reentrant superconductivity. The example of such behavior for FM/SC bilayer is shown in the Fig. 7, taken from Ref. [23]. However, for a complete calculations the requirement of a finite interface transparency has also

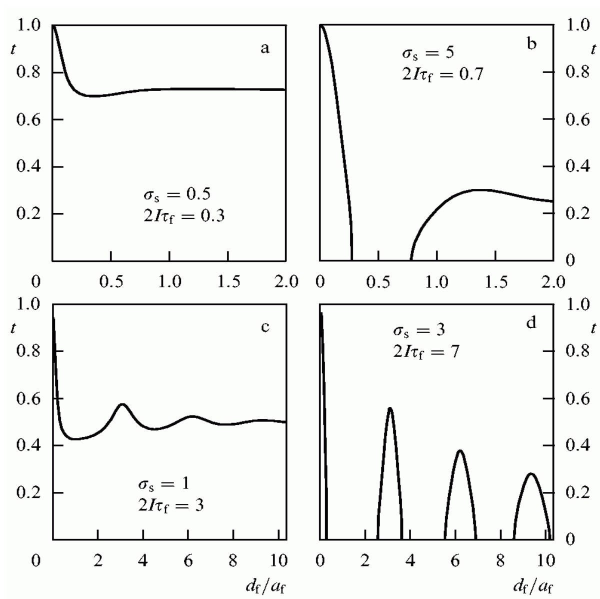

Figure 7: Reduced transition temperature $t = T_c/T_{BCS}$ as a function of the reduced thickness of the FM layer for different strengths of disorder τ_f (from a) strongest disorder to d) cleanest system). σ_S is the interface transmittance and $I \equiv E_{ex}$. Plot a) represents emergence of T_c onto a plateau; b) reentrant superconductivity; c) oscillations of T_c; d) periodically reentrant superconductivity. The dashed curves t^* in parts b and d represent lines of tricritical points. Adapted from Ref. [23].

to be assumed.

As it was mentioned, the bulk $FFLO$ state also features spontaneously generated currents, flowing in the ground state. Similar currents have been recently predicted [34, 80, 81] to occur in FM/SC proximity effect. Such currents flow, depending on the exchange splitting and the thickness of FM layer. The origin of these currents and the conditions for their flowing are closely related to the Andreev bound states in FM/SC heterostructures [34, 80, 81] and will be discussed in Sec. 7.

5 Origin of Andreev bound states

From quasiclassical considerations, each bound state corresponds to particle moving along a family of closed trajectories [97]. The energy of such bound state is determined by the Bohr-Sommerfeld quantization rules, according to which the total phase accumulated during one cycle has to be equal to multiples of 2π. Interestingly, the bound states also emerge in the normal metal/superconductor (NM/SC) structures [31] due to the Andreev reflections [3], according to which an incident electron is reflected back as a hole at the interface, and a Cooper pair is created in SC. Such states are built up from a combination of electron and hole wave functions. The example of the closed quasiparticle trajectory, producing the bound state, in an insulator/(normal metal)/superconductor $I/NM/SC$, is shown in the Fig. 8. It consists of an

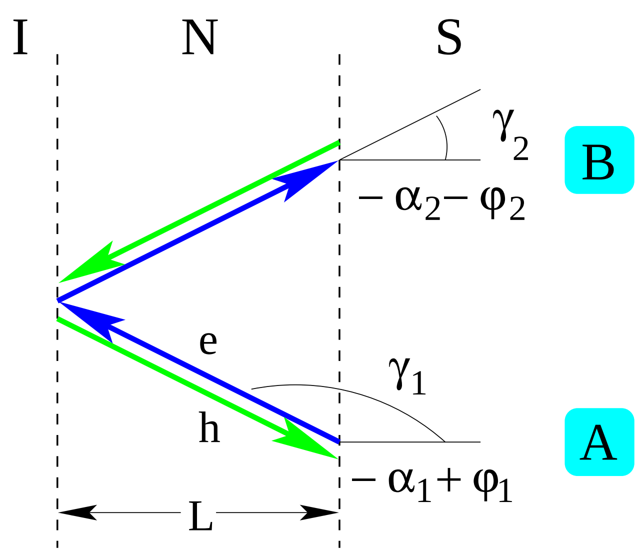

Figure 8: The example of the quasiparticle path corresponding to the Andreev reflections, giving a bound state. The quasiparticle is trapped in the normal region because of normal reflection at the I/NM surface and the Andreev reflection at the NM/SC interface. The total phase accumulated during one cycle is equal: $-(\alpha_1 + \alpha_2) \pm (\varphi_1 - \varphi_2) + \beta(E)$.

electron e segment, which includes a ordinary reflection at the I/NM interface, and hole h one, retracing backwards the electron trajectory. The total accumulated phase in this case consists of contribution from Andreev reflections at point A: $-\alpha_1 + \varphi_1$ and B: $-\alpha_2 + \varphi_2$ as well as contribution from the propagation through the normal metal $\beta(E)$. $\alpha_{1(2)} = arccos(E/|\Delta_0|)$ is the Andreev reflection phase shift, while $\varphi_{1(2)}$ is the phase of the SC order parameter at point A (B). $\beta(E) = 2L(k_e - k_h) + \beta_0$ is the electron-hole dephasing factor and describes the phase acquired during the propagation through the normal region, where the first term corresponds to the ballistic motion and the second one to the reflection at the I/NM surface. L is the thickness of NM, and k_e (k_h) is the electron (hole) wave vector. Thus the Bohr-Sommerfeld quantization condition is:

$$-(\alpha_1 + \alpha_2) \pm (\varphi_1 - \varphi_2) + \beta(E) = 2n\pi \tag{1}$$

where the $\pm(\varphi_1 - \varphi_2)$ stands for the trajectories in the $\pm k_y$ (parallel to the interface) direction.

If there is no phase difference between points A and B in the Fig. 8, as for instance in the case of the (normal metal)/(s-wave superconductor) interface, Eq. (1) gives the energies of the bound states in the form [31]:

$$\frac{E}{\Delta_0} = \pm cos\left(\frac{2EL}{\Delta_0 \xi_0 cos(\gamma_2)}\right) \tag{2}$$

where $\xi_0 = \hbar v_F/\Delta_0$ is the SC coherence length and γ_2 is the angle between electron trajectory and the surface normal (see Fig. 8). The number of such states for each quasiparticle trajectory is determined by the length of the quasiparticle path, which in turn is given by the NM thickness and the propagation angle γ_2.

As it can be read from Eq. (2), the bound states always appear in pairs symmetrically positioned around the Fermi level because of the time reversal symmetry in the problem. Moreover, due to the fact that there is no difference between electrons and holes at the Fermi level ($\beta(E = 0) = 0$), there is no $E = 0$ solution. In other words, the bound states always emerge at finite energies.

The situation is quite different if there is a phase difference ($\varphi_1 - \varphi_2$) between points A and B (see Fig. 8). The example can be the interfaces with d-wave superconductors oriented in the (110) direction, where ($\varphi_1 - \varphi_2) = \pi$. In this case, due to the additional phase shift π, the Eq. (1) has the solution:

$$\frac{E}{|\Delta_0|} = \pm sin\left(\frac{2EL}{|\Delta_0| \xi_0 cos(\gamma_2)}\right) \tag{3}$$

From this it follows that bound states can emerge even at zero energy. Such zero-energy Andreev bound states, in the case of high-T_c superconductors, have been predicted by Hu [32] and are known as *zero-energy mid-gap states.* The presence of the Andreev bound states at zero energy features in many important effects, like zero-bias conductance peaks, π-junction behavior, anomalous temperature dependence of the critical Josephson current, paramagnetic Meissner effect, time reversal symmetry breaking and spontaneous interface currents [29, 30].

Although the zero-energy states (ZES) are likely to appear when the phase of the order parameter at the interface is not constant, the resulting density of states at the Fermi energy is energetically unfavorable and any mechanism able to split these states will lower the energy of the system [30, 98]. On of these is the self-induced Doppler shift [99, 29] $\delta = ev_F A$, where A is a vector potential. The situation is schematically depicted in the Fig. 9. At low temperature ($T^* \approx (\xi_0/\lambda)T_c$, where λ is the penetration depth of the magnetic field) the splitting of the zero energy states produces a surface current. This current generates a magnetic field (screened by a supercurrent), which further splits ZES due to the Doppler shift effect. The effect saturates when the magnetic energy is equal to the energy of the Doppler shifted ZES.

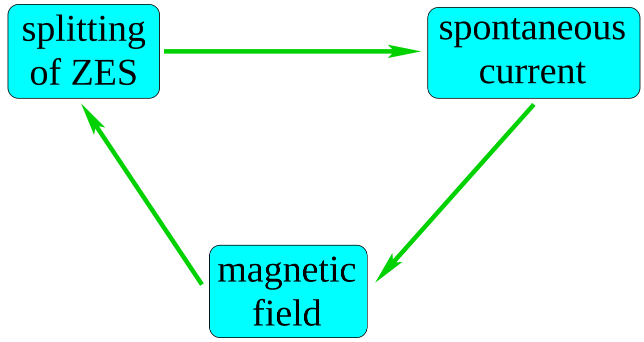

Figure 9: Generating of the spontaneous currents.

6 Andreev bound states in I/FM/SC trilayer

Naturally, the Andreev bound states also arises in $I/FM/SC$ heterostructures [33, 100, 83, 76, 94, 80, 81]. More importantly, as it was first predicted by Kuplevakhskii & Fal'ko [33], it is possible to shift these states to zero energy by tuning the exchange splitting. Again, the energies of such states can be obtained from the quasiclassical arguments (Eq. (1)) with modified electron-hole dephasing factor $\beta(E)$ due to the exchange splitting. Thus the solution of the Eq. (1) now is:

$$\frac{E_\sigma}{\Delta_0} = \pm cos\left(\frac{1}{2} + cos(\varphi_1 - \varphi_2) + \frac{\sigma\pi L}{2cos(\gamma_2)\xi_{FM}}\right) \tag{4}$$

Clearly, the crossing of the zero energy solution can be obtained either by changing the phase difference $(\varphi_1 - \varphi_2)$ or by varying FM coherence length (exchange field). The energies of the bound states as a function of the reduced FM thickness L/ξ_{FM} is shown in the Fig. 10. It is

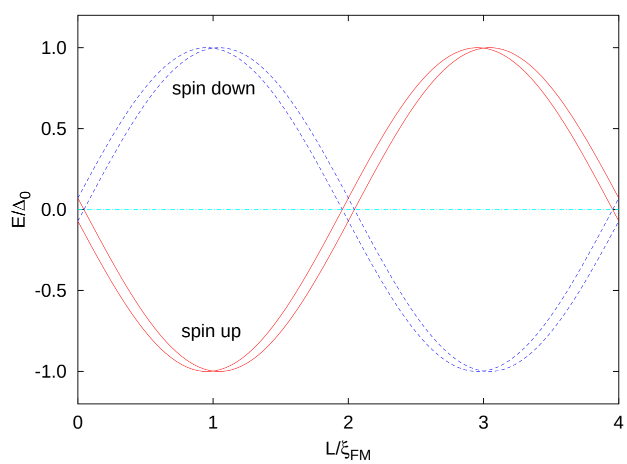

Figure 10: Positions of the Andreev bound states E/Δ_0 as a function of the reduced FM thickness L/ξ_{FM} for $\gamma_2 = 0$ and $(\varphi_1 - \varphi_2) = 0$ (see Fig. 8). The energies obtained from Eq. (4).

worthwhile to note that crossing the zero energy appears periodically, and also depends on the angle of the particle incidence γ_2 (see Fig. 8).

The properties of such bound states have been also studied fully quantum-mechanically within lattice models of the FM/SC systems [94, 80, 81] and similar their behavior have been obtained. Interestingly, it turns out, that as in the case of the high-T_c structures [99], such zero energy Andreev states support spontaneous currents flowing in the ground state of the FM/SC system [34, 80, 81]. The mechanism of generating of such currents is the same, as earlier discussed, namely the self-induced Doppler shift. So in fact, when the current flows, such one of the states will be twice shifted: once due to the exchange (Zeeman) splitting, and the second time due to the Doppler shift. Schematically, the situation is depicted in the Fig. 11.

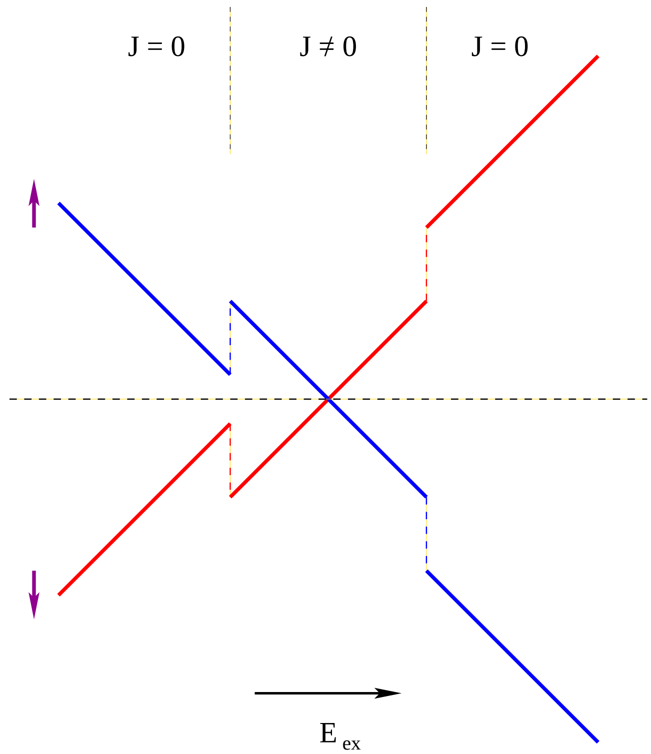

Figure 11: The effect of the spontaneous current on the positions of Andreev bound states. Despite usual (Zeeman) splitting ($J = 0$), there is also Doppler shift due to the current flowing ($J \neq 0$).

For energies less than superconducting gap, the only Andreev bound states will contribute to the density of states $\rho(E)$. However, as it was mentioned, for fixed thickness and exchange splitting, there will be Andreev bound states at different energies, for different angles of particle incidence (γ_2 in the Fig. 8). Thus to get the density of states, one has to sum the energies of these states over all values of γ_2:

$$\rho(E) = \sum_{\gamma_2=-\pi/2}^{\pi/2} \delta(E - E_{bound}) \tag{5}$$

and talk, in fact, about Andreev bands rather that single states. However, all that was said on properties of the bound states, remains true for Andreev bands too. In particular the splitting of the whole band due to the spontaneous current is ilustrated in the Fig. 12. The additional structure comes from the other (higher order) Andreev reflections. Superconducting energy gap $\Delta_0 = 0.376$ in this figure.

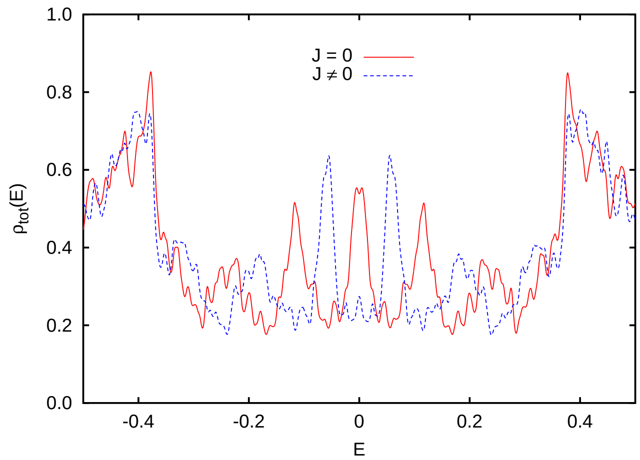

Figure 12: Doppler splitting of the zero-energy state. From Ref. [80].

There is also a strong correlation between Andreev bound states (bands) and the pairing amplitude [94, 34, 80]. Each time the pairing amplitude at the I/FM interface changes its sign, the Andreev bound state (band) crosses the Fermi energy. Moreover in this case the spontaneous current is generated.

7 Spontaneous currents

One of the most remarkable properties of the ferromagnet/superconductor proximity systems is the appearance of spontaneous currents flowing in the ground state. Such currents have been recently predicted [34, 80, 81] in fully self-consistent treatment of the $I/FM/SC$ trilayers within a simple tight binding Hubbard model. These currents appear even there is no any external magnetic filed and, as it could be expected, in the case of ferromagnet, they are spin-polarized under certain conditions.

7.1 Linear current response

As it was mentioned, the large density of states at the Fermi level is energetically unfavorable, so the spontaneous current is generated thus lowering the energy of the system. The appearance of the spontaneous currents can be explained very easily within linear current response theory [101]. According to this the total current can be divided into two parts: diamagnetic one giving a response of the bulk density and the paramagnetic one, which is due to the deformation of the wave function at the Fermi surface and thus proportional to the density of states at the Fermi level (at $T = 0$). So if $\rho(E_F) = 0$, the paramagnetic current vanishes, and nothing is happening when there is no external magnetic field. On the other hand, if there is a sharp peak at E_F, this gives rise to the paramagnetic current, which overcompensates the diamagnetic one and thus leads to the instability and creation of the spontaneous current. So in fact one can say that spontaneous current is proportional to the density of states at the Fermi level.

7.2 Fully self-consistent treatment

The spontaneous current appears in a natural way when the FM/SC system is treated self-consistently within the lattice tight bounding Hubbard model [34, 80, 81]. In this case a number of equations for SC order parameter, spin polarization, chemical potential, current and its polarization as well as a vector potential has to be solved fully self-consistently. Details of calculations, which fully confirm the above picture, can be found in [80].

The typical example of such current, flowing parallel to the FM/SC interface, is shown in the Fig. 13. The current flows mostly in positive y direction on ferromagnetic side and in negative

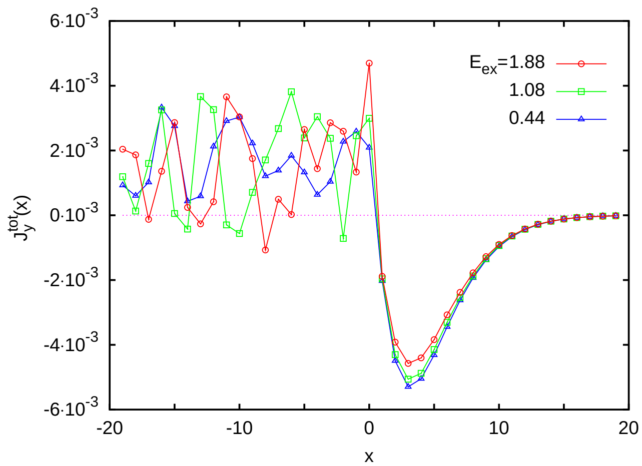

Figure 13: Spontaneous current (in units of $et/\hbar$) flowing parallel to the FM/SC interface for different exchange splittings. From Ref. [80].

direction in superconductor. The total current, integrated over the whole sample, is equal to zero, as it should be for the ground state according to the Bloch theorem. Moreover, the current carrying by quasiparticles exactly cancels the supercurrent [80], as in the bulk $FFLO$ state [18]. In the case of FM/SC system one would expect that supercurrent should flow mainly in superconductor while quasiparticle one in ferromagnet due to the spatial separation of the ferromagnetism and superconductivity. However, it turns out, that this is not the case, one cannot completely spatially separate them. Neveretheless the cancellation does not occur layer by layer and hence locally there are spontaneous currents and magnetic field.

7.3 Spontaneous magnetic field

Obviously, the spontaneous current distribution (see Fig. 13) generates the magnetic field through the sample. The total magnetic flux weakly depends on the thickness of the sample and the exchange splitting. Its magnitude is found to be a fraction of the flux quantum $\Phi_0 = h/2e$ and is smaller than upper critical field of the bulk superconductor. This is rather a large field and could be observable.

Such magnetic field can be used to detect the spontaneous currents. In particular its temperature dependence [81], shown in the Fig. 14. It is worthwhile to note that spontaneous magnetic flux appears below SC transition temperature T_c, at $T^* \approx (\xi_0/\lambda)T_c$. The fact that T^* and T_c are

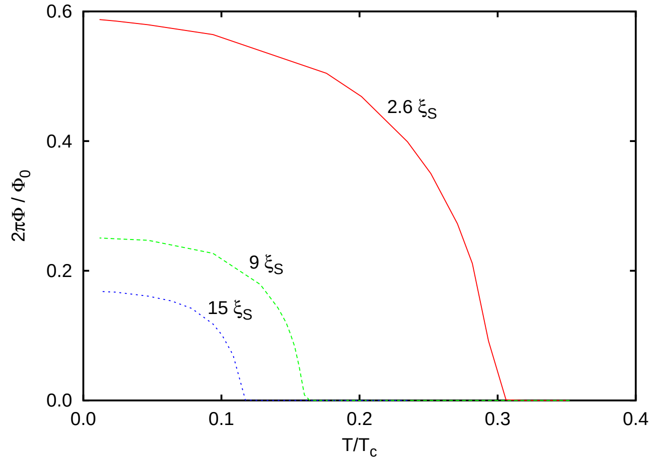

Figure 14: The temperature dependence of the total magnetic flux for different thicknesses of the FM region. From Ref. [81].

different temperatures may help to unambiguously confirm the existence of the spontaneous currents in FM/SC heterostructures.

7.4 Current modified density of states

Owing to the fact that spontaneous currents flow in whole ferromagnet, the splitting of the Andreev bands can be seen in the surface (I/FM) density of states [81], which in turn can be directly measured experimentally [25]. The temperature dependence of the surface density of states at the Fermi energy $\rho(\varepsilon_F)$ (see Fig. 12) can also give a clear indication of such spontaneous current flowing in the system. When the current flows in the system, there is a huge drop in the

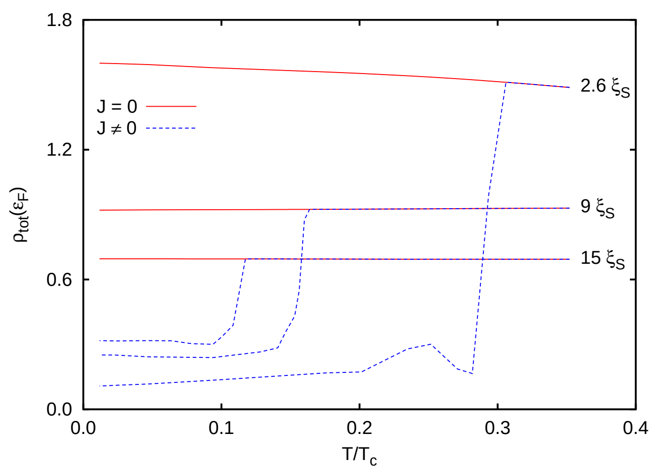

Figure 15: The temperature dependence of the surface I/FM density of states at the Fermi energy. The dashed (solid) line corresponds to the situation in which there is (there is no) current. From Ref. [81].

$\rho(\varepsilon_F)$ at certain temperature $T^* = (\xi_S/\lambda)T_c$, which is usually much smaller that SC transition temperature T_c. Of course such drop in the DOS is caused by the Doppler splitting of the Andreev band.

7.5 Polarization of the current

In the FM/SC proximity system one would expect the spontaneous current to be polarized. It turns out, this is the case only for the ferromagnet with different concentration of electrons and holes, i. e. away from the $e-h$ symmetry point. This can be explained as follows: As it has been discussed at beginning of this section, the spontaneous current is proportional to the density of states at the Fermi energy. In the case of the particle-hole symmetry the density of states for spin up and spin down electrons are the same, regardless the exchange splitting. So as there are no differences in the spin up and spin down DOS, there will not be polarization of the current [34, 80]. Of course in reality, there is no $e-h$ symmetry in ferromagnet, so one would expect non-zero polarization of the current.

7.6 2D FFLO state

As it was mentioned earlier, at certain conditions a $3D$-$FFLO$ state is energetically more favorable than usual $1D$ state. Furthemore, changing the thickness of the FM slab, one can switch the ground state of the system between $3D$ and $1D$-$FFLO$ state [67, 6]. From above self-consistent calculations one can conclude that the same effect emerges in natural way if one considers the spontaneous current in the system. The argument is as follows: The oscillations of the pairing amplitude in the direction perpendicular to the interface occur regardless the spontaneous current. The period of them is related to the x-component of the center of mass momentum of the Cooper pair in FM $\mathbf{Q} = (2E_{ex}/v_F)\frac{\mathbf{v}_F}{v_F}$. This can be interpreted as the usual $1D$-$FFLO$ state in confined geometry, refered to in Sec. 4. On the other hand, in $2D$ geometry studied here, when the current flows parallel to the interface, there is a finite vector potential in the y-direction. This can be regarded as a y-component of the $\mathbf{Q}$-vector. So one can say that when the spontaneous current flows, the $2D$-$FFLO$ state is realized. Moreover when the FM thickness is changed the ground state of the system is switched between $2D$- and $1D$-state, which manifests itself in spontaneous current flow or its lack. Clearly this behavior is consistent with the findings of Izyumov *et al.* [67, 6].

8 Conclusions

The competition between ferromagnetism and superconductivity in FM/SC heterostructures give raise to the Fulde - Ferrell - Larkin - Ovchinnikov ($FFLO$) state in these systems. The original bulk $FFLO$ state manifests itself in a spatial oscillations of the SC order parameter as well as in spontaneously generated currents flowing in the ground state of the system. I have argued that a very interesting version of this phenomenon accures in FM/SC proximity systems. In short, due to the proximity effect and the Andreev reflections at the FM/SC interface, the Andreev bound states appear in the quasiparticle spectrum. These states can be shifted to the zero energy by tuning the exchange splitting or the thickness of the ferromagnet, thus they became zero-energy mid-gap states which lead to various interesting effects. It particular, the spontaneous currents can be also related to the zero-energy states, as in the case of high-T_c superconductors. It seems that some combination of both phenomena is realized in a real systems. The fact that oscillatory behavior of SC order parameter is strongly correlated with the crossing of the Andreev bound states through Fermi energy and the generation of the spontaneous currents further support $FFLO$ - Andreev bound states picture.

There is also a strong experimental evidence that $FFLO$ - Andreev bound states scenario is really realized in FM/SC structures. This includes the SC transition temperature and the oscillations of the density of states as the thickness of the FM slab is changed. The observation of the $FFLO$ state in such systems is probably related, unlike in the bulk, to its non-sensitivity to a disorder, as it has been suggested theoretically. Additionally, the experimental confirmation of the existence of the spontaneous (spin polarized) currents in the ground state will support the $FFLO$ - Andreev bound states scenario in these structures.

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